**Module 1 Discussion: Discrete and Continuous Probability:**

1. Select a discrete probability distribution, and provide a real-life example of one or more applications of that distribution. Explain how your example matches the conditions for the particular distribution that you have selected.
2. Select a continuous probability distribution, and provide a real-life example of one or more applications of that distribution. Explain how your example matches the conditions for the particular distribution that you have selected.

In one of the replies to a classmate, mention another application of the discrete random variable and continuous random variable that your classmate has chosen. Describe similarities and contrasts between your example and your classmate's example.

**Discrete Example:**

* A grocery store owner wants to find out the number of customers the store serves within a day. Using this distribution, the owner can find the probability of customers visiting the store within a day
* Mean is the average of all the observed values when the number of customers visiting the store within a day. Standard deviation measures how spread out the number of customers visiting the store in a day is
* Suppose the average number of customers within a day is 300 and the restaurant owner wants to find out the probability of serving 400 customers in a day. This distribution can be used to calculate if the store is serving enough customers daily or not so that he can plan to pre-load the stock and increase the staff accordingly. He can also look for store expansions and the revenues it generating from them. Hence, computing these attributes can help the store owner predict the expected revenue and customer visits on particular days

**Continuous Example:**

* The lipozene experiment is a continuous normal distribution. For example, let’s calculate the probability that a person lost weight between 20 to 40 pounds after taking the pill. With this distribution, we can calculate average weight loss of fewer than 20 pounds, greater than 20 pounds, or between 20 to 40 pounds, but the probability that someone lost exactly 40 or 20 pounds would be zero
* Mean is defined by the density curve of the distribution. If it is a normal distribution curve, then the mean would lie in the center of the curve. So, the probability of weight loss is within a range of values. And here the mean of a weight loss is a measure of the center of the distribution. Also, the standard deviation would be the measure of the dispersion of the weight lost between 20 and 40 pounds
* This is used to test out the effectiveness of lipozene. This information, in particular, can help the scientists to test out and provide proof for their claim

**References:**

* List of Probability Distributions from Wikipedia, <https://en.wikipedia.org/wiki/List_of_probability_distributions>
* Probability Distributions: Discrete vs. Continuous, <https://stattrek.com/probability-distributions/discrete-continuous.aspx>

**Module 2 Discussion: Sampling, Estimation, and Simulation:**

1. Explain how Monte Carlo simulation is used in the real world. Provide a specific example from your line of work, or a line of work that you find particularly interesting.
2. Identify the parts or aspects of Monte Carlo simulation processes that you have found to be particularly challenging. Describe why you believe that they are challenging and provide remedies to simplify those aspects.

In your two replies to classmates, provide remedies to simplify those aspects of the Monte Carlo simulation processes that they have found to be challenging.

**Monte Carlo Simulation:**

It showcases all the likely outcomes of our decisions and measures the impact of risk. These are used to model the probability of different outcomes in a process that cannot easily be predicted due to the intervention of random variables. It is a technique used to understand the uncertainty in prediction and forecasting models

**Sampling, Estimation, & Optimization:**

* Sampling is to gather information about a random object by observing many insights into it
* Estimation is to estimate certain numerical quantities related to a simulation model. An example is the estimation of the expected throughput in a production line
* Optimization in Monte Carlo techniques are used to optimize noisy functions, where the function itself is random

**Real-world Application:**

As financial products continue to grow in complexity, Monte Carlo techniques have become increasingly important tools for analyzing them. These techniques are also used in risk analysis, scenario analysis, and pricing basket options on a portfolio of stocks

**Challenges:**

* How accurate are the results/ outcomes?
* Identifying the Risks and potential consequences
* Computational costs and complexity of integration
* Be cautious with the Convergence Rates and Randomized Algorithms

**Remedies:**

* An appreciation of the overall degree of variability and uncertainty and the confidence that can be placed in the analysis and its findings
* Understanding the key sources of variability, critical assumptions, and their impacts on the analysis
* Identify and manage the risks that may impact these activities

**References:**

[1] Palisade, Monte Carlo Simulation was retrieved from https://www.palisade.com/risk/monte\_carlo\_simulation.asp

[2] Christopher Pease (Sep 6, 2018), An Overview of Monte Carlo Methods was retrieved from https://towardsdatascience.com/an-overview-of-monte-carlo-methods-675384eb1694

**Module 3 Discussion: Regression and Forecasting:**

In many applications, a time series decomposition (i.e., time-series filtering) is used to separate or decompose a time series Xt into its trend, seasonal, and irregular components. In some of these applications, the decomposition relationship is assumed to be additive (Xt = Trendt + Seasonalt + Irregulart); while in other applications, the decomposition relationship is assumed to be multiplicative (Xt = Trendt x Seasonalt x Irregulart).

1. Explain the merits of such decomposition methods, and mention a particular example of a time series where you believe that implementing a decomposition technique is justified. Explain your reason(s) for selecting such an example.
2. Explain in what situations you would prefer to use an additive decomposition method, and in what situations you would prefer to use a multiplicative method in your time series decomposition.

**Regression:**

We need to forecast the time series of y assuming that it has a linear relationship with other time series x. In this, y is the dependent variable and x is the independent variable.

**Forecasting:**

It uses the information in a time series to forecast future values of that series.

**Example:**

Let’s assume we toss a fair coin 10 times and calculated its outcomes.

* The coin will have one of two outcomes either head or tail. The probability that a head occurs is 0.5 and that a tail occurs is again 0.5
* In this scenario, the tossing of the coin each time is independent of each other  
  If an occurrence of one side of a coin is a success, then the other side is failure
* It follows a binomial distribution with 10 and 0.5 as parameters and the probability of success p = 0.5 and probability of failure = q = 1 – p = 1 - 0.5 = 0.5

**Time-series Decomposition:**

Time series are full of patterns and relationships. Decomposition intends to find and segregate them into different components, each with definite properties and behavior. Let us consider airline passenger data from 1950 to 2020. There was a boom in the number of people taking the airplane over time. It has the linear trend with seasonality to be in an increasing manner.

**Additive Decomposition Method:**

It is used when the seasonal variation is comparatively constant over time. In this, the terms seem to add together leading to the time-series. In this, the components are independent amounts. **Ex:** Sales of a restaurant overtime

**Multiplicative Decomposition Method:**

It is used when the seasonal variation rises over time. In this, the terms seem to be more exaggerated over time. Also, it has components that are proportional to fluctuation in data. **Ex:** An increase in Users on the Web (Traffic).

**References:**

[1] Wikipedia, Decomposition of time series was retrieved from https://en.wikipedia.org/wiki/Decomposition\_of\_time\_series

[2] Additive vs Multiplicative model in Time Series Data was retrieved from https://datascience.stackexchange.com/questions/46050/additive-vs-multiplicative-model-in-time-series-data

**Module 4 Discussion: Decision Modeling & Risk Analysis:**

A decision model may be descriptive, heuristic, or prescriptive.

* In your judgment, what are some important requirements for making good decision models regardless of whether they are descriptive, heuristic, or prescriptive? How would you check the validity of a model that you have selected?
* Describe the reasons why a manager might use a heuristic decision model instead of a prescriptive model. Use a real-life example, and explain your reasons within the context of your example.

**Decision Models:**

These are used to understand, analyze, and facilitate in making a decision. It represents a real object and has some relationships. They contain data, decisions, and uncontrollable variables.

**Types of Decision Models:**

* **Descriptive:** It describes the relationships and provides information for evaluation. It involves building and testing formal models that improve the descriptive capability
* **Heuristic:** These are simple strategies or mental processes that are used to quickly form judgments, make decisions, and find solutions to complex problems. This happens when an individual focuses on the most relevant aspects of a problem or situation to formulate a solution. It includes using trial and error, a rule of thumb, or an educated guess
* **Prescriptive:** It is the optimization models which tend to define an optimal policy and make the best decision to maximize/ minimize the objective. Provides the guidelines in uncertain situations to make decisions.

**Requirements for Making Good Decision Models:**

* Define a Problem and Identify your Goal
* Set criteria to make possible decisions with proper data/ information
* Set the importance of each criterion and analyze the costs
* What are the possible alternatives and Evaluate them?
* Determine the best way to address the problem and make a decision

**Model Validation:**

It can be done by Continuous Tracing and Testing. One approach to assessing the quality of your model and separate the dataset between a test set and a training set. The initial analysis is done on the training data. Those results are then applied to the test data to determine the validity. Assessing prediction uncertainty, quality, and reliability of the model. Send the results to the business stakeholder, get some feedback, and determine the results. Also, focus on Physical observations and calibration checks.

**Reasons to use a Heuristic Decision Model:**

* They are used to take quick judgmental decisions, find solutions, see help, and analyze complex problems. Elimination, Recognition, Take-the-best, and Fast-and-Frugal Trees are some of the strategies and procedures in the Heuristic Decision Model
* Managers usually make decisions by reflecting on action or having an understanding only of the immediate situation and surface suitable courses of action
* Decision-makers identify the best decision to take by computing, with perfect accuracy, how different decision alternatives will play out. They make choices that are based on assumptions and gut feelings about the future
* They summarize it, recognize patterns of similarity between the new situation and what they had experienced or learned, by filling in missing details based on previous experiences and make assumptions
* Intuition plays a significant role in managers’ daily work-life especially when decisions need to be made quickly or unexpectedly because potential costs are associated with delays, or because of a high level of uncertainty and insufficient information
* It is an important strategy that managers rely on in their decision-making process and it helps them improve the organization’s performance, especially in an unstable environment
* When managers become aware of the potential adverse impact of using heuristics, they become capable of deciding when and where to use them and eliminating certain heuristics from their decision-making collection

**References:**

[1] J. van der Pligt (2001) Decision Making, Psychology of, was retrieved from https://www.sciencedirect.com/topics/computer-science/human-decision-making

[2] Jonathan Baron (24th Dec 2012) The point of normative models in judgment and decision making was retrieved from https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3529390/

**Module 5 Discussion: Linear, Integer & Binary Programming**

The development of linear programming is perhaps among the most important scientific advances of the mid-20th century. Today, it is one of the standard tools that has saved many millions of dollars for most companies and enterprises.

1. Identify at least two reasons for the importance of linear programming in an enterprise of your choice and describe the impact that linear programming has had in that enterprise in recent decades.
2. Provide a specific example of a linear programming model related to the enterprise that you have selected and interpret the slack variables of your example.
3. Describe how the information obtained about the slack variables of your example can be used by the decision-making sector of that enterprise. (You do not have to write a mathematical formulation of your example. Simply mention the objective, the decision variables, and a few possible constraints to describe your model example)

**Linear Programming:**

It is an optimization technique for a system of linear constraints and a linear objective function. An objective function defines the quantity to be optimized, and the goal is to find the values of the variables that maximize/ minimize the objective function.

**Examples:**

* It could be applied to manufacturing, to calculate how to minimize the cost of operations
* It could be applied in high-level business operations to maximize profit
* It could also be applied in logistics to get a job done in the minimum amount of time

**Slack Variables:**

These represent the positive difference, or slack, between the left-hand side of the inequality and the right-hand side of that inequality. These are included in the simplex algorithm when converting the constraints and objective functions into a system of equations

**Enterprise Application:**

Let’s assume I own a Chocolate company and I can produce/ manufacture 2 varieties of chocolates i.e., milk and dark chocolate. The ultimate goal in any business is to satisfy the customers and make some profits. With the data that I have and projections made so far, I can see that there will be a demand of > 500 lbs of milk chocolate and 400 lbs of dark chocolate daily. We can make a profit of $1 on white chocolate and $1.5 on dark chocolate. So, how much should we need to sell to maximize our profit?

1. **Importance:**

* It helps in attaining the optimum use of resources and materials
* It gives an idea to make decisions by effective selection and distribution of resources
* It improves the quality of decisions and provides the best feasible solution
* It helps in analyzing the supply chain, operations, storage, and delivery routes

1. **Impacts:**

* Formulating a problem and limiting the choices on decision variables
* Understanding Linear programming in assessing a problem
* Resource Utilization

1. **Slack Variables:**

We need to add two different slack variables to the two different inequalities. Slack variables represent the number of excess resources. Ordinarily, they have a zero objective function coefficient and a plus one entry in a single constraint. We need to check which resources are left unused. In this, we can identify electricity, resources, materials, and storage as some of the slack variables.

1. **Decision Variables:**

These are the unknown quantities that are expected to be estimated as an output.

* Let, P1 = No. of Milk Chocolate production daily
* P2 = No, of Dark chocolate production daily

1. **Objective Function:**

The objective is to attain a maximum profit. The selling price of Milk chocolate is $5 and dark chocolate is $6. So, based on these, we can calculate the unit profit made by these chocolates

1. **Constraints:**

The constraints are the restrictions or limitations on the decision variables.

* No more than 400 lbs of milk and 250 lbs of dark chocolates can be produced daily
* A shipping total should be about 400 lbs daily

**References:**

[1] Introduction to the simplex method was retrieved from https://www.matem.unam.mx/~omar/math340/simplex-intro.html

[2] The Editors of Encyclopedia Britannica, Linear Programming was retrieved from https://www.britannica.com/science/linear-programming-mathematics

**Module 6 Discussion: Non-linear Programming**

A Quadratic programming model is an optimization model whose objective function has a quadratic form. Furthermore there are n decision variables and m linear constraints in the model . Mathematically, a Quadratic programming problem has the following form:

Minimize Z = 1/2 xT Q x + CT x

Subject to: A x ≥ b, x ≥ 0.

Here, x is the n by 1 column vector of decision variables and xT is its transpose, Q is an n by n symmetric matrix of objective parameters, C is an n by 1 vector of additional objective parameters, A is an m by n matrix of constraints parameters, and b is a m by 1 vector of constraints right hand side. The factor (1/2) is only a scaling factor, and is mathematically insignificant.

1. Explain how quadratic programming is used in the real world. Provide a specific example from your own line of work, or a line of work that you find particularly interesting. Indicate explicitly and qualitatively what Z, x, Q, C, A, and b are in your example.
2. Harry Markowitz, and Myron Scholes along with Robert Merton are the Nobel Laureates in Economics in 1990 (Markowitz) and 1996 (Scholes and Merton) respectively. Markowitz won the Nobel award for devising his Modern Portfolio Theory (also called: the Markowitz Portfolio Theory - MPT) in 1952. Scholes and Merton were the recipients of Nobel for their Option Pricing and Volatility models introduced in early 1980s. Explain how each of the above two models (MPT and Option Pricing volatility analysis) are related to Quadratic programming. Describe the decision variables, the objective function, and the constraints for each model.

In your two replies to classmates describe instances, scenarios and conditions under which their Quadratic programming model example may not have (i) a feasible solution; (ii) a unique solution.

**Quadratic Programming:**

* It has an objective which is a quadratic function of the decision variables, and constraints which are all linear functions of the variables. An example of a quadratic function is: X12 + 2 X22 + 3 X1X2 where X1, X2 and X3 are decision variables
* Markowitz mean-variance portfolio optimization problem is mainly used where the quadratic objective is the portfolio variance, and the linear constraints specify a lower bound for portfolio return
* These problems have only one feasible region with flat faces on its surface due to the linear constraints, but the optimal solution may be found anywhere within the region or on its surface
* The quadratic objective function may be convex. Non convex and the problem becomes simple/ complex. Portfolio optimization problems are usually of this type
* This has applications in finance, computer systems, statistics, production, and in algorithms to solve more complex Non linear programming problems
* The solver is capable of finding optimal solutions for positive definite or semi-definite quadratic objectives when minimizing, negative definite/ semi-definite when maximizing

**Option Pricing Volatility:**

* Quadratic Programming is a type of non-linear programming model with limits as constraints are imposed on the result. It is all about finding the vector z that minimizes a quadractic function
* Purchasing options is a strategy defined to limit risk and increase potential profits. The value of an option depends on the futire volatility of a stock rather than on its expected return. With the option investing, our initial loss is limited to the amount spent on purchasing the option
* Then, we can buy/ sell the stock. Here, the decision variable is whether the option required to buy/ sell the stock. So, it will create an overall profit/ loss
* Objective function is to maximize the Profit spending the least amount of money. One constraint could be whether they can short the stock/ not. The investor might limit the options they open at given time. Or, he might trade only at certain price

**Markowitz’s Portfolio Theory:**

* It is regarding maximizing the returns that an investor could get or not considering the risk. It asks the investor to consider how much the risk of one investment can impact the entire portfolio
* With a well-balanced and calculated portfolio, if some of the assets fall due to market conditions, others should rise an equal amount in compensation, according to MPT
* It is a great example of Quadratic Programming and decision variable can be the level of risk a portfolio takes. There are other several parameters that can exist as well
* So, given a choice between a higher return possibility with greater risk, and a lower return possibility with less risk, most people will naturally prefer the portfolio with the least risk, even if it means a lower return
* The investor then naturally views a correlation between increased risk and potential higher returns as compensation

**References:**

[1] Jerome Bracken, Garth P. McCormick (Oct 1968) Selected Applications of Nonlinear Programming was retrieved from https://apps.dtic.mil/dtic/tr/fulltext/u2/679037.pdf

[2] Daniel Ahlbom, (June 2017) Quadratic Programming Models in Strategic Sourcing Optimization was retrieved from https://uu.diva-portal.org/smash/get/diva2:1159097/FULLTEXT01.pdf